POLARIZATION OF THE CMB PHOTONS CROSSING THE GALAXY CLUSTERS

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Abstract:

In this paper, I investigate a local effect of polarization of the Cosmic Microwave Background (CMB) in clusters of galaxies, induced by the Thomson scattering of an anisotropic radiation. A local anisotropy of the CMB is produced by some scattering and gravitational effects, as, for instance, the Sunyaev Zel'dovich effect, the Doppler shift due to the cluster motion and the gravitational lensing. The resulting anisotropy $\Delta I/I$ depends on the physical properties of the clusters, in particular their emissivity in the X band, their size, their gravitational potential and the peculiar conditions characterizing the gas they contain. By solving the Boltzmann radiative transfer equation in presence of such anisotropies I calculate the average polarization at the centre of some clusters, namely A2218, A576 and A2163, whose properties are quite well known. I prove that the gravitational effects due to the contraction or to the expansion have some importance, particularly for high density structures; moreover, the peculiar motion of the cluster, considered as a gravitational lens, influences the propagation of the CMB photons by introducing a particular angular dependence in the gravitational anisotropy and in the scattering integrals. Thus, the gravitational and the scattering effects overall produce an appreciable local average polarization of the CMB, may be observable through a careful polarization measurements towards the centres of the galaxy clusters.

arXiv:astro-ph/9701203v1 27 Jan 1997

1. INTRODUCTION

The first detection of the CMB anisotropy, performed by COBE (Smoot et al. 1992) four years ago, opened a large debate about the presence of a detectable polarization in the background radiation: in fact, one expects (Rees 1968) that the Thomson scattering of such an anisotropic radiation on the electrons plasma produces a linear polarization.

The CMB propagation is generally studied by using the Boltzmann transfer equation (Peebles 1980); the Chandrasekhar formalism (Chandrasekhar 1960) is particularly useful to study a polarization problem as the one I want to handle. After the introduction of a scattering matrix that depends on the polar angles (θ', ϕ') of the incoming beam, the evolution of the radiation with the time, the photon energy and with the space coordinates is described by using a partial derivative differential equation, containing, as variables, the four Stokes parameters fixing its polarization status at all the times.

One should distinguish between a *global* and a *local* effect of CMB anisotropy and polarization: the global anisotropy is produced by some primordial density fluctuations (see, for instance, Harari & Zaldarriaga 1993) and by some time-dependent fluctuations of the space-time metric (i.e. by gravitational waves, originated, for instance, during the inflationary era (Sachs & Wolfe 1967).

I discussed these effects, particularly those due to tensor perturbations, in (Gibilisco 1995; Gibilisco 1996); similar approaches to this problem can be found in (Basko & Polnarev 1980; Polnarev 1985; Crittenden et al. 1993a and 1993b).

Here, I want to study the CMB transfer equation from a *local* point of view, i.e. considering the anisotropy effects in clusters of galaxies: in this case, the clusters properties determine in a fundamental way the characteristics of the CMB photons which cross them. In particular, the possible sources of CMB anisotropy in cluster of galaxies are the gravitational bound (GB) effect, the collapse (C) or expansion (E) effect, the gravitational lensing (GL), the Sunyaev Zel'dovich (SZ) effect and finally a Doppler-like shift (D): all these effects will be discussed below.

In this work I prove the importance of the gravitational phenomena: in particular, the motion of the cluster, considered as a lens, introduces an additional angular dependence in the scattering integral, thus conditioning the cancellation of some terms there; the gravitational collapse effect dominates for massive clusters, having a high density, while the Doppler shift is always negligible if compared to the SZ effect; note however that it may have some importance in a different framework, namely in galactic halos: in this case, one expects a total irrelevance of the SZ and of the gravitational effects, due to the low values of density and temperature characterizing the halos; only a significant fraction of MACHOS within them could enhance the relevance of the gravitational phenomena, but more theoretical investigations are necessary to clarify this problem.

Here, through the mathematical solution of the Boltzmann transfer equation, I calculate the overall CMB polarization induced by the Thomson scattering of this locally anisotropic radiation and I evaluate the relative importance of these effects. In the following, for simplicity, I will assume to have a spherical shape for the clusters and an isothermal model for the gas they contain (see, for instance, Boynton et al. 1982).

This paper is structured as follows: in sect. 2 I will briefly discuss the theory of the radiative transfer, by recalling the main equations governing the photons propagation in presence of Thomson scattering; I will also recall the formalism of the Stokes parameter describing the polarization status of the CMB. In sect. 3 I will discuss in some detail the peculiar effects contributing to create a local anisotropy for the CMB crossing the clusters; their dependence on the polar coordinates will be examined and the consequences of such a behaviour will be pointed out. In sect. 4 I will solve the transfer equation by studying the space evolution of the Stokes parameters on time-constant hypersurfaces; then, I will show my results for the average polarization degree in some clusters, namely A2218, A576 and A2163. Finally, in sect. 5 I will present my conclusions, emphasizing the importance of the gravitational effects in producing a local polarization of the CMB.

2. THE THEORY OF THE RADIATIVE TRANSFER

A detailed discussion of the radiative transfer theory in presence of Thomson scattering was given by Chandrasekhar (Chandrasekhar 1960); the fundamental equation governing the evolution of the photon distribution function is the Boltzmann equation (for a detailed explanation see, for instance, (Peebles 1980)) mathematically expressing the Liouville theorem of the conservation of the probability density in the phase space.

A suitable way to write the Boltzmann equation adopts as variables the four Stokes parameters, locally describing the polarization status of the radiation (Chandrasekhar 1960); these parameters are used as components of a 4-vector $n_\alpha = n_\alpha(\theta, \phi, \nu)$, which is a function of polar angles (θ, ϕ) and of the photon frequency ν:

$$n_\alpha \equiv (I_l, \; I_r, \; U, \; V). \tag{2.1}$$

In eq. (2.1) I_l, I_r are the left and right intensities, defined as in the Chandrasekhar geometrical formalism (Chandrasekhar 1960) based on the polarization ellipse (see fig. 1); with this definition, the total intensity of the radiation is $I = I_l + I_r$, while $Q = I_l - I_r$ and U are the parameters indicating the presence of a linear polarization; finally, V represents the circular polarization, but its evolution can be neglected because it totally decouples from the others parameters (see Appendix). As a result, the Thomson scattering of an anisotropic radiation produces *linear* polarization only, as proved by Rees in (Rees 1968).

The final expression for the radiative transfer is a partial differential equation that reads:

$$\left(\frac{\partial \vec{n}}{\partial \eta} \; + \; \gamma^\alpha \; \frac{\partial \vec{n}}{\partial x^\alpha} \right) \; + \; \frac{\partial \nu}{\partial \eta} \; \frac{\partial \vec{n}}{\partial \nu} \; = \; \frac{\sigma_T \; N_e \; R(\eta)}{4\pi} \; \times$$

$$\times \left[\; - 4\pi \vec{n} \; + \; \int_{-1}^{1} \int_{0}^{2\pi} \vec{n}(\mu', \phi') \; P(\mu, \phi, \mu', \phi') \; d\mu' \; d\phi' \; \right]. \tag{2.2}$$

In eq. (2.2), η is the comoving time, defined as $\int [dt/R(t)]$, and $R(t)$ is the scale factor of the Universe; γ^α are the components of an unit vector in the propagation direction of the photons, $\sigma_T = 6.65 \times 10^{-25} \; cm^2$ is the Thomson scattering cross-section, $\mu = \cos \theta$ and $P(\mu, \phi, \mu', \phi')$ is the Chandrasekhar scattering matrix, whose explicit form as a function of the polar angles is given in (Chandrasekhar 1960) and partially shown in the Appendix;

finally, N_e is the comoving number density of free electrons, depending on the ionization history of the Universe.

An analytical way to solve the transfer equation in presence of cosmological gravitational waves has been discussed in (Gibilisco, 1995) on the basis of the theory of the Volterra integral equations: here it is sufficient to recall that eq. (2.2) contains a collisionless part, given by the time component of the geodesic equation, and a collisional term, represented by the integral in its right-hand side. That really gives all the informations about the scattering processes, responsible for the production of a linearly polarized, outgoing radiation.

3. LOCAL EFFECTS PRODUCING THE CMB ANISOTROPY AND POLARIZATION IN CLUSTERS OF GALAXIES

Two kinds of local effects in galaxy clusters may produce an anisotropy and, consequently, a polarization in the CMB crossing these structures: the first class consists of gravitational effects, due to the distortion of the Hubble flow in the region of a massive clusterl; the latter one is due both to the inverse Compton scattering of the photons on the plasma electrons and to a Doppler-like shift, connected with the motion of the structure. In the following I will recall in some detail all these effects, pointing out the local CMB anisotropy they induce in order to determine the level of linear polarization generated by the Thomson scattering.

a) **Gravitational effects:**

1) *Moving Gravitational lenses:*

A massive object always influences the radiation that propagates in the surrounding region, gravitationally distorting its path; in particular, a source observed behind a massive structure appears slightly different in frequency and flux if compared to a similar one located in a different position: that represents a typical lensing phenomenon.

A similar, but less known, effect is produced when the object representing the gravitational lens is *in motion* across the line of sight (Birkinshaw 1989); in this case, the lens produces a) an *anisotropy* in the CMB crossing the lens and b) a redshift difference between the multiple images of a background object seen through it.

These effects have been discussed in (Birkinshaw 1989; Birkinshaw & Gull 1983); here we are interested in the production of an anisotropy in the CMB, a phenomenon that depends on the transverse velocity of the lens and presents a peculiar angular dependence.

Looking at fig. 2a, we consider the CMB as seen through a convex lens (for instance a galaxy cluster), moving with a velocity $\vec{v}$ which forms an angle α with the line of sight: δ is the deflection angle of the radiation and θ is the angle between the deflected direction of propagation and the line of sight (see fig. 2b) (Birkinshaw & Gull 1983). In the lens frame, the frequency of the radiation is unchanged: on the contrary, for a stationary observer in the frame of the original source, the observed frequency of the deflected light depends on

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$\vec{v}$ and δ as follows (Birkinshaw & Gull 1983):

$$\frac{\Delta\nu}{\nu} \;=\; \gamma\,\beta\,\delta\,\sin\alpha\,\cos\phi; \tag{3.1}$$

here β and γ respectively are v/c and the Lorentz factor, while ϕ is the angle between the lens velocity vector and the emerging ray projected onto the sky (see fig. 2b). The deflected photons have different energies and different brightness in comparison with the undeflected ones: as a result, the corresponding change in the brightness temperature of the radiation field is (Birkinshaw & Gull 1983):

$$\frac{\Delta T}{T}(\theta,\phi,\alpha) \;=\; \left(\frac{8\gamma\beta GM_0\sin\alpha}{c^2 L}\right)\cos\phi\cdot\frac{\theta_0}{\theta}\left[1-\left(1-\frac{\theta^2}{\theta_0^2}\right)^{3/2}\right]; \tag{3.2}$$

eq. (3.2) holds for $\theta \leq \theta_0$; M_0 is the lens mass (that, for a galaxy cluster, is about $10^{15}\,M_\odot$), L is the average cluster radius ($L \sim 3\,Mpc$) and θ_0 is given by the ratio L/d where d is the cluster distance.

From eq. (3.2) one can remark that the CMB anisotropy and the induced polarization could be relevant for cluster of galaxies having large transverse peculiar velocities: in fact, in a rich cluster of galaxies, the deflection angle due to the lensing is relatively large ($\sim 1\,arcmin$), while the transverse velocity may be larger than $1000\,km\,s^{-1}$ (Dressler 1987).

2) *Effects due to the gravitational bound:*

The decoupling of a massive, gravitationally bounded structure from the local Hubble flow produces a redshift in the radiation that crosses it, when compared to the one that passes far away; in fact, the metric of the massive object changes in a different way as regards the one of the surrounding Universe. As a result, a monochromatic source observed behind a massive structure will be observed at a shifted frequency compared to a similar source placed in a different position.

This redshift effect in a variable gravitational field was theoretically studied by(Rees & Sciama 1968; Dyer 1976; Nottale 1984) in the framework of a "Swiss cheese" model: this model starts from a zero-pressure Robertson-Walker Universe where one removes a comoving sphere of dust and places a clump of the same mass at the centre of the "hole" thus obtained. In this way, the optical effects of the massive object are influenced by the surrounding void in the embedding Universe: that is due to the localized changes of the space-time metric in the region where the inhomogeneity is present and where the distortion of the Hubble flow is significant.

In this model, a cluster is represented by a high density Friedmann solution of the Einstein equations, separated by an empty Schwarzschild zone from the surrounding, lower density Friedmann Universe (Nottale 1984); the radiation crossing both the empty Schwarzschild zone and the massive central structure suffers two effects (Dyer 1976): a) a redshift effect, implying that the parameters $1 + z \;=\; \nu/\nu_0$ and $1 + \xi \;=\; R(T_0)/R(T)$ (R is the scale factor) do not coincide behind the cluster; b) a time-delay effect, due to the

fact that the radiation crossing the inhomogeneous structure follows a longer path. For the CMB photons the temperature anisotropy resulting from these effects reads (Nottale 1984):

$$\frac{\Delta T}{T} = -4 \ q_0 \ \frac{\overline{\rho_c}}{\rho_0} \ \left(\frac{H_0 r}{c}\right)^3 \left(1 - \frac{H_c}{H_0} - \ln\left(\frac{\overline{\rho_c}}{\rho_0}\right)^{1/3}\right), \qquad (3.3)$$

where q_0 is the deceleration parameter, L is the cluster size, $\overline{\rho_c}$ and ρ_0 are the average density of the cluster and the one of the background Universe and finally H_c and H_0 are the Hubble constants for the cluster and for the background Universe at the present time. The ratio $\overline{\rho_c}/\rho_0$ is about equal to 10 in the case of an expanding cluster, while it is about 1000 for a collapsing structure (Nottale 1984).

3) *Effects due to the gravitational collapse or expansion:*

A phenomenon very similar to the one above discussed is produced when the change of the metric is due to the collapse or to the expansion of the massive object (Nottale 1984): a rich cluster may both contract ($H_c/H_0 < 0$) or expand ($H_c/H_0 > 0$); however, the observations seem to support the possibility of a collapse (Capelato et al. 1982; de Vaucouleurs 1982).

The resulting temperature anisotropy of the CMB photons is (Nottale 1984):

a) for a contracting cluster ($H_c/H_0 < 0$):

$$\frac{\Delta T}{T} = -5 \left[q_0 \ \frac{\overline{\rho_c}}{\rho_0} \ \left(\frac{H_0 r}{c}\right)^2 \right]^{3/2}; \qquad (3.4)$$

b) for a slightly expanding cluster ($0 < H_c/H_0 \le 1$):

$$\frac{\Delta T}{T} = -4 \ q_0 \ \frac{\overline{\rho_c}}{\rho_0} \ \left(\frac{H_0 r}{c}\right)^3 \left(q_0 \ \frac{\overline{\rho_c}}{\rho_0} - \ln\left(\frac{\overline{\rho_c}}{\rho_0}\right)^{1/3}\right); \qquad (3.5)$$

c) for a rapidly expanding cluster ($H_c/H_0 > 1$):

$$\frac{\Delta T}{T} = 4 \ q_0 \ \frac{\overline{\rho_c}}{\rho_0} \ \left(\frac{H_0 r}{c}\right)^3 \frac{H_c}{H_0}; \qquad (3.6)$$

These gravitational effects seem to dominate the scattering ones; that is surely true for very massive clusters ($M > 10^{19} \ M_\odot$), as proved in (Rees & Sciama 1968); in the following I will prove their relevance also for lower masses ($M \sim 10^{14} - 10^{15} \ M_\odot$).

b) **The Sunyaev-Zel'dovich and the Doppler effects:**

The Sunyaev-Zel'dovich effect (SZ) (Sunyaev & Zel'dovich 1972) consists in a characteristic distortion of the CBM photons spectrum due to the inverse Compton scattering of the radiation on the hot electron gas ($T \sim 10^8 \ K$) present in the clusters (Lea et al. 1973). That produces a shift in the CMB spectrum towards smaller wavelengths near the

blackbody peak at $\lambda \sim 1\ mm$ and a consequent decrement of the radiation intensity in that region; correspondingly, one observes an increment of the intensity at submillimetric wavelengths.

This effect has been studied at centimeter wavelengths in many clusters, as Abell 2319 (White & Silk 1980) Abell 576 (Boynton et al. 1982), (White & Silk 1980), Abell 2218 (Boynton et al. 1982; Klein et al. 1991; Jones et al. 1993), Abell 665 and Cl0016+16 (Birkinshaw et al. 1984); a significant evidence of the SZ distortion has probably been obtained for Abell 2218 (Boynton et al. 1982). In fact, the properties of the core gas in A576 and in some other clusters desagree with the theoretical expectations for the inverse Compton scattering process and therefore the apparent CMB decrement could be due to some spurious effects (White & Silk 1980). Anyway, the presumable average magnitude of the temperature decrement is in the range $3 \times 10^{-5} \div 10^{-3}\ K^{\circ}$ (White & Silk 1980); for Abell 2218 Klein et al. found $\Delta T = -0.21 \pm 0.04\ mK^{\circ}$ at $\lambda = 1.2\ cm$ (Klein et al. 1991), while Jones et al. (Jones et al. 1993) found $\Delta T \sim 0.5\ mK^{\circ}$.

From a theoretical point of view, the evaluation of the magnitude of the SZ effect depends on the cluster model one assumes, in particular on the X ray emissivity, on the cluster size and mass, on the form of the gravitational potential and on the physical conditions characterizing the core gas. Generally, one supposes the hot gas is in an isothermal and quasi-hydrostatic equilibrium and the cluster is a stationary and spherically symmetric structure (White & Silk 1980).

The X-ray emission mainly comes from the bremsstrahlung processes and it is given by (White & Silk 1980):

$$\Lambda(T) \ = \ 3.2 \times 10^{-24} \left[\ \ln\ \left(1 + \frac{0.13T}{E_1 + E_2}\right) \ + \ 2.51 \Big(\frac{E_1 + E_2}{T} \ + \ 0.5\Big)^{-1/2} \ \right] \times$$

$$\times\ T^{1/2} \left[\ exp(-E_1/T) \ - \ exp(-E_2/T) \ \right]\ ergs\ cm^3\ s^{-1}, \tag{3.7}$$

where the passband is (E_1, E_2) and T is the gas temperature, all expressed in KeV. The emissivity profile is given by (White & Silk 1980):

$$L(r) \ = \ n_e^2(r)\ \Lambda(T) \ = \ \frac{n_e^2(r)}{2}\ \frac{l_0}{r_c}\ \left(1\ +\ \frac{r^2}{r_c^2}\right)^{-3/2}, \tag{3.8}$$

where l_0 and r_c are the central surface brightness and the X-ray core radius, known from the observations.

Finally, in an isothermal model for the gas (Boynton et al. 1982; Klein et al. 1991) the density profile is given by the following formula:

$$n_e \ = \ n_{e,0}\ \left(1\ +\ \frac{r^2}{r_c^2}\right)^{-3n/2}; \tag{3.9}$$

where $n_{e,0}$ is the central electron density, equal to $(3.4 \pm 1.5) \times 10^{-3}\ cm^{-3}$, and n is the power law index, equal to 0.5 ± 0.1 (Klein et al. 1991).

For a gas whose properties are given by eqs. (3.6)-(3.9), the microwave decrement at the centre of the cluster in the Rayleigh-Jeans limit is (White & Silk 1980):

$$\frac{\Delta T_M}{T_M} = -\frac{2k\sigma_T}{m_e c^2} (2r_c l_0)^{1/2} \cdot \int_0^\infty T(x)(\Lambda[T(x)])^{-1/2} (1+x^2)^{-3/4} dx, \qquad (3.10)$$

where $\sigma_T = 6.65 \times 10^{-25}$ cm^2 is the Thomson scattering cross section and the microwaves temperature has been called T_M in order to distinguish it from the gas temperature T; finally, $x = h\nu/kT$.

In an isothermal model with $T = 12.5$ KeV the integration of eq. (3.10) gives (White & Silk 1980):

$$\frac{\Delta T_M}{T_M} = -7.4 \frac{k\sigma_T}{m_e c^2} (r_c l_0)^{1/2} T [\Lambda(T)]^{-1/2}; \qquad (3.11)$$

eq. (3.11) is the final expression I will assume for the SZ temperature decrement.

An additional decrement is due to the peculiar motion of the cluster (Sunyaev & Zel'dovich 1972 and 1980): that is just a Doppler-like effect and it causes a change of the radiation intensity and of the CMB temperature depending on the radial component v_r of the peculiar velocity of the cluster (Sunyaev & Zel'dovich 1980):

$$\frac{\Delta T_M}{T_M} = -\frac{v_r \tau}{c}, \qquad (3.12)$$

$$\frac{\Delta I}{I} = -\frac{x \, exp(x)}{(exp(x) - 1)} \frac{v_r \tau}{c}; \qquad (3.13)$$

here τ is the optical depth, equal to $\tau = \int_{-L}^{L} n_e(r)\sigma_T \, dr$.

While the thermal SZ effect gives the same perturbation both for the temperature and the intensity, in this case the expressions for $\Delta T/T$ and $\Delta I/I$ are different: in particular, the perturbed intensity only (eq. (3.13)) depends on the frequency. This effect should be dominant in galactic halos, where the gravitational and thermal contributions probably are negligible.

4. THE CMB POLARIZATION IN A2218, A576, A2163.

Now I can apply the formalism developed in sects. 2 and 3 to study the CMB polarization in some observed clusters.

The clusters considered in this analysis were studied in many observations and their properties are quite well determined; moreover, they are possible candidates for which the observation of the Sunyaev-Zel'dovich decrement has been claimed.

In tabs. 1a, 1b and 1c I resume the physical properties of these clusters useful for my calculation of the CMB polarization.

4.1 THE CLUSTERS A2218, A576, A2163.

Cluster A2218

Following the original Abell catalogue, A2218 is classified as a richness class 4, distance class 6 cluster. Subsequently, its redshift has been fixed to 0.174 (Le Borgne et al. 1992) and its class fixed as Bautz-Morgan II (Leir & Van den Bergh 1978); it contains a large cD galaxy and many weak radio sources (Andernach et al. 1988); a review of the present characteristics of A2218 can be found in (Boynton et al. 1982; Klein et al. 1991).

Cluster A576

A576 is a moderately rich cluster, classified as Abell richness class 1, distance class 2 and Bautz-Morgan class III (Leir & Van den Bergh 1978); it presents a central condensation region but none dominant galaxy; the cluster contains a high proportion of SO galaxies (Melnick & Sargent 1977), it is surrounded by an extensive X-ray emitting halo (Forman et al. 1978) but it is a quite weak X-ray source. A review of its properties can be found in (White & Silk 1980).

Cluster A2163

A2163 is a rich cluster of Rood and Sastry class I (Struble & Rood 1987) (corresponding to $N_{Abell} = 119$ galaxies) having a redshift $z = 0.201$. The spectroscopic X-ray observations, performed with the GINGA satellite (Arnaud et al. 1992) showed an exceptionally high temperature and X-ray luminosity ($kT = 13.9^{+1.1}_{-1.0}$ KeV and $L_X = 6.0 \times 10^{45}$ erg/s in the band $2 - 10$ KeV, while the usual values for other clusters are respectively lower than 9 KeV and 2×10^{45} erg/sec). Due to the fact that the SZ effect is proportional to the product of the electron density and of the temperature, A2163 surely represents a very promising candidate for the observation of the SZ microwave decrement. A review of the present experimental knowledge of A2163 can be found in (Elbaz et al. 1995).

4.2 THE CALCULATION OF THE POLARIZATION

Before calculating the CMB polarization induced by the previously discussed anisotropies we should firstly fix a suitable reference frame to work. If we look at figs. 2a and 2b, a possible choice consists in fixing the origin of the frame in coincidence with the observer on the earth; the z axis is oriented towards the centre of the cluster, along the line of sight. For simplicity, I assume an approximate spherical symmetry for the clusters, a hypothesis in most cases well confirmed by the observations.

I introduce the usual polar coordinate system (r, θ, ϕ) and I define $\theta_0 = L/d$, where L is the cluster radius and d is its distance from the observer. Then, I write the transfer equation (2.2) in this particular reference frame. Before the scattering Thomson the anisotropic radiation is unpolarized: the perturbed intensity reads

$$[\Delta I]_{tot} = [\Delta I(\theta, \phi, \alpha)]_{Lens} + [\Delta I(r)]_{Grav.b.+coll./exp.} + [\Delta I(T)]_{SZ} + [\Delta I(\nu, v_r)]_{Doppl},$$
$$(4.2.1)$$

i.e. it is given by the sum of the various contributions previously discussed. The incoming radiation in the transfer equation is represented by a vector $\vec{n}_{in}$ containing the perturbed

intensity:

$$\vec{n}_{in} \equiv (\Delta I/2, \ \Delta I/2, \ 0). \tag{4.2.2}$$

(as usual, I neglect the Stokes parameter V because no circular polarization is produced). After the Thomson scattering on the freee eleectron plasma, the anisotropic radiation acquires a linear polarization (Rees 1968) and the vector expressing its status is:

$$\vec{n}_{scatt} \equiv (\Delta I_l, \ \Delta I_r, \ \Delta U). \tag{4.2.3}$$

Then, the transfer equation in polar coordinates reads:

$$3 \ \frac{\partial \vec{n}_{scatt}}{\partial r} \ + \ \frac{1}{r} \cdot \frac{\partial \vec{n}_{scatt}}{\partial \theta} \ [2 \tan \theta \ - \ \cot \theta] \ + \ \frac{1}{r} \cdot \frac{\partial \vec{n}_{scatt}}{\partial \phi} \ [\tan \phi \ - \ \cot \phi] \ =$$

$$-\sigma_T \ n_e(r) \ R(\bar{t}) \ \vec{n}_{scatt} \ + \ \frac{\sigma_T}{4\pi} \ R(\bar{t}) \ n_e(r) \ \int_0^{2\pi} \int_{\mu_0}^1 d\mu' \ d\phi' \ P(\mu, \mu', \phi, \phi') \ \vec{n}_{in}; \tag{4.2.4}$$

($\mu_0 = \cos \theta_0$; the primed angular variables refer to the incoming radiation).

Eq. (4.2.4) holds on fixed-time hypersurfaces: the reference time $\bar{t}$ corresponds to the observed redshift of the cluster. The variable θ in the integration spans over the range $(0, \theta_0)$.

4.3 THE RESULTS

The CMB polarization is obtained by solving eq. (4.2.4): due to the axial simmetry of the problem, this expression simplifies, because the term containing the ϕ derivative vanishes.

Substituting the vectors (4.2.2) and (4.2.3) into eq. (4.2.4), I obtain a system of three partial differential equations in the perturbed parameters ΔI_l, ΔI_r, ΔU, whose solution represents the final polarization status of the CMB in the cluster. The system reads as follows:

$$3 \ \frac{\partial \Delta I_l}{\partial r} \ + \ \frac{1}{r} \ \frac{\partial \Delta I_l}{\partial \theta} \ (2 \tan \theta \ - \ \cot \theta) \ = \ \sigma_T \ n_e(r) \ R(\bar{t}) \ \left[\ \frac{I_0}{8\pi} \ R_1(\theta, \phi, r, \nu, \alpha, T) \ - \ \Delta I_l \ \right],$$
$$\tag{4.3.1a}$$

$$3 \ \frac{\partial \Delta I_r}{\partial r} \ + \ \frac{1}{r} \ \frac{\partial \Delta I_r}{\partial \theta} \ (2 \tan \theta \ - \ \cot \theta) \ = \ \sigma_T \ n_e(r) \ R(\bar{t}) \ \left[\ \frac{I_0}{8\pi} \ R_2(\theta, \phi, r, \nu, \alpha, T) \ - \ \Delta I_r \ \right],$$
$$\tag{4.3.1b}$$

$$3 \ \frac{\partial \Delta U}{\partial r} \ + \ \frac{1}{r} \ \frac{\partial \Delta U}{\partial \theta} \ (2 \tan \theta \ - \ \cot \theta) \ = \ \sigma_T \ n_e(r) \ R(\bar{t}) \ \left[\ \frac{I_0}{8\pi} \ R_3(\theta, \phi, r, \nu, \alpha, T) \ - \ \Delta U \ \right].$$
$$\tag{4.3.1c}$$

In eqs. (4.3.1a), (4.3.1b) and (4.3.1c) I called R_1, R_2 and R_3 the scattering integral that appears in eq. (4.2.4), while $R(t)$ is the scale factor of the Universe; I separate the different contributions to the anisotropy in the following way:

$$\left(\frac{\Delta I}{I} \right)_{tot} \ = \ F_1(\theta', \phi', \alpha) \ + \ F_2(r) \ + \ F_3(\nu, T), \tag{4.3.2}$$

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where F_1 is given by eq. (3.2) and represents the lensing contribution (the primed angular variables must be used because we are referring to the incoming radiation) F_2 is given by eq. (3.3) plus one of the eqs. (3.4), (3.5) or (3.6), depending on the cluster evolution (gravitational bound + collapse or expansion term) and finally F_3 is given by the sum of eq. (3.11) (thermal SZ effect) and eq. (3.13) (Doppler effect); then the scattering integrals read ($j = 1, 2, 3$):

$$R_j(\theta, \phi, r, \nu, \alpha, T) \;=\; \int_0^{2\pi} \int_{\cos\theta_0}^1 d\mu' \, d\phi' \, (a_{j1} + a_{j2}) \left[F_1(\theta', \phi', \alpha) \;+\; F_2(r) \;+\; F_3(\nu, T) \right].$$

(4.3.3)

In eq. (4.3.3), a_{j1} and a_{j2} are the matrix elements of the Chandrasekhar scattering matrix, whose form can be found in Appendix.

A possible way to perform the calculation consists in searching a recursive solution of eqs. (4.3.1a), (4.3.1b) and (4.3.1c): I expand the perturbations in Legendre polynomials as follows:

$$\Delta I_l \;=\; \frac{I_0}{r} \sum_l a_l \, P_l(\cos\theta),$$

(4.3.4a)

$$\Delta I_r \;=\; \frac{I_0}{r} \sum_l b_l \, P_l(\cos\theta),$$

(4.3.4b)

$$\Delta U \;=\; \frac{I_0}{r} \sum_l c_l \, P_l(\cos\theta),$$

(4.3.4c)

where I_0 is the unperturbed intensity, given by:

$$I_0 \;=\; \frac{1}{exp(h\bar{\nu}/k_B T) - 1};$$

(4.3.5)

here $h\bar{\nu} \sim 10^{-13} \, GeV$ and $k_B T \sim 2.35 \times 10^{-13} \, GeV$. Then, it is possible to calculate the coefficients of the expansion simply by solving the differential equations (4.3.1a), (4.3.1b) and (4.3.1c); the recursive calculation can be stopped at $l = 2$, because the higher order terms in the polynomials are negligible. Finally, the polarization degree P is

$$P(r, \theta, \phi) \;=\; \frac{|\,\Delta I_l - \Delta I_r + \Delta U\,|}{(\Delta I_l + \Delta I_r)}.$$

(4.3.6)

In tabs. 2a, 2b and 2c I show a comparison of the different contributions F_1, F_2 and F_3 to the local CMB anisotropies for the considered clusters and in the cases of a contracting, slightly expanding or totally expanding cluster; the data listed in these tables have been calculated at some reference values for the polar coordinates and for an angle $\alpha = \pi/2$ in order to maximize the lensing contribution. In tab. 3 I list the values of the coefficients which appear in the expansions (4.3.4a), (4.3.4b) and (4.3.4c) of the perturbed Stokes parameters for the three clusters: I show the values obtained for fixed (r, θ, ϕ) in order to give an idea of the size of these coefficients. Finally, in figs. 3, 4 and 5 I plotted the

average polarization degree in the clusters A2218, A576 and A2163 as resulting from this calculation.

5. DISCUSSION AND CONCLUSIONS

Looking at eqs. (4.3.1a), (4.3.1b) and (4.3.1c), we remark that the different evolution of the perturbed Stokes parameters originates in the scattering integrals, as an effect of the dissimilarity of the functions R_1, R_2 and R_3.

The angular dependence of the matrix elements a_{j1}, a_{j2} (see eqs. (A.1), (A.2) and (A.3) of the Appendix) and of the gravitational lensing term F_1 plays a fundamental rôle in determining the Stokes parameters evolution and, therefore, in producing a CMB polarization; for instance, the R_3 scattering integral vanishes when integrated on ϕ': as a result, all the coefficients c_l of the expansion (4.3.4c) are equal to zero and the overall contribution of ΔU to the polarization cancels. A linear polarization results from the scattering contributions R_1 and R_2: the difference existing between these functions causes an unequal evolution of the perturbed left and right intensities and therefore an unvanishing term $|\Delta I_l - \Delta I_r|$ in eq. (4.3.6).

In tabs. 2a, 2b and 2c, I considered both the possibilities of a collapse or an expansion for the clusters: we can remark that the most important contribution to the CMB anisotropy and polarization is due to the gravitational collapse of the clusters (F_2 term); the high densities attained in this case strongly enhance the importance of the gravitational effects but, anyway, also for an expanding cluster F_2 dominates. Moreover, the SZ effect (F_3 contribution) dominates over the lensing one (F_1 contribution), while the Doppler shift is totally negligible at the considered frequencies; note however that the lensing contribution slightly influences the angular behaviour of the scattering integral, by introducing an additional $\cos \phi'$ dependence.

In tab. 3 I listed the coefficients of the expansion in Legendre polynomials, obtained by solving the differential equations (4.3.1a), (4.3.1b) and (4.3.1c): the recursive calculation can be stopped at the order $l = 2$ because the coefficients vanish very fast.

These results confirm that the gravitational effects are important in determining the characteristics of the CMB photons crossing the galaxy clusters, particularly in the case of very massive and dense structures. In figs. 3, 4 and 5 I showed the behaviour of the average polarization degree in galaxy clusters as a function of the polar angle θ: due to the smallness of $\theta_{max} = \theta_0 = L/d$, this dependence is very weak, indicating that an observation performed towards the clusters centres should be able to measure the CMB polarization induced by these effects. Possible fluctuations in the signal might be attributable to the presence of some central radiosources which introduce a serious source of noise.

The average polarization degree for A2218 and A2163 is comparable, while for A576 it is smaller by a factor 10: that is probably due to the z dependence of the gravitational bound effect.

No relevant differences are found between the polarization in expanding or in collapsing clusters: that is due to the fact that a change in the function F_2 affects in the same way the values of the left and right perturbed intensities, thus leaving unchanged the quantity $|\Delta I_l - \Delta I_r|$.

Summarizing, in this paper I studied the CMB polarization in clusters of galaxies induced by local anisotropies: by expanding the Stokes parameters in Legendre polynomials and by solving the Boltzmann transfer equation through a recursive method, I proved that the gravitational effects have some importance: they indeed influence the properties of the CMB photons crossing the galaxy clusters. The cancellation of some terms in the Thomson scattering integral makes $|\Delta I_l - \Delta I_r|$ different from zero, while the resulting perturbation in the Stokes parameter U vanishes.

In such a way, the Thomson scattering of the locally anisotropic CMB radiation (the anisotropy being due both to the scattering and to the gravitational effects) produces an appreciable polarization in clusters having a mass $M \geq 10^{14} \div 10^{15} \ M_{\odot}$ An accurate observation towards the clusters centres might, in principle, reveal such a phenomenon, provided that one is able to distinguish the contribution due to the possible presence of strong radiosources.

APPENDIX

The Chandrasekhar scattering matrix (Chandrasekhar 1960) is a 4×4 matrix having the following structure:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & 0 \\ a_{21} & a_{22} & a_{23} & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ 0 & 0 & 0 & a_{44} \end{pmatrix}$$

The presence of only one matrix element different from zero in the position (4,4) assures the Stokes parameter V is totally decoupled from the other ones: as a result, the Thomson scattering of the anisotropic radiation does not produce a circular polarization.

When we apply the scattering matrix to the vector (4.2.2), representing the incoming radiation, we need to know the following linear combinations of the matrix elements a_{ij}:

for eq. (4.3.1a):

$$a_{11} + a_{12} = \frac{3}{4} \left[3\mu^2 \mu'^2 - \mu^2 - 2\mu'^2 + 2 + 4\sqrt{1-\mu^2} \sqrt{1-\mu'^2} \, \mu\mu' \, \cos(\phi' - \phi) + \right.$$

$$\left. + \mu^2 \mu'^2 \cos 2(\phi' - \phi) - \mu^2 \cos 2(\phi' - \phi) \right]; \qquad (A.1)$$

for eq. (4.3.1b):

$$a_{21} + a_{22} = \frac{3}{4} \left[1 + \mu'^2 + \cos 2(\phi' - \phi) \, (1 - \mu'^2) \right]; \qquad (A.2)$$

for eq. (4.3.1c):

$$a_{31} + a_{32} = \frac{3}{2} \left[-2\mu' \sqrt{1-\mu^2} \sqrt{1-\mu'^2} \, \sin(\phi' - \phi) + \sin 2(\phi' - \phi) \, (\mu - \mu\mu'^2) \right]. \quad (A.3)$$

The ϕ' dependence of eqs. (A.1), (A.2) and (A.3) produces the cancellation of some terms in the scattering integrals; the anisotropy due to the lensing effect (see eq. (3.2)) also contains a factor $\cos \phi'$, thus, combining these dependences, I obtain from eq. (4.3.3) $R_3 = 0$ and $R_1 \neq R_2$. This difference between R_1 and R_2 is finally responsible for the creation of a local polarization in clusters.

A grateful thank goes also to all the people of the Astrophysical Section of the University of Milano; in particular, I would like to address an especial thank to Silvio Bonometto, who firstly gave me the idea to investigate about this problem and then the way to pursue this work. Finally, I want to address the most affectionate thank to my friends, Emma and Francesco.

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Tab. 1a: Properties of the cluster Abell 2218 (Boynton et al. 1982; Klein et al. 1991).

CLUSTER PROPERTIES	DATA
z	0.174
d	$1060 \pm 810\ Mpc$
L	$\sim\ 3\ Mpc$
θ_0	2.83×10^{-3}
r_c	$0.22 \pm 0.06\ Mpc$
l_0	$1.60 \times 10^{-4}\ erg\ cm^{-2}\ sec^{-1}$
n_0	$(4.2 \pm 1.8) \times 10^{-3}\ cm^{-3}$
x	$0.438\ (\nu = 25\ GHz)$
M_T	$(7.8 \pm 1.4) \times 10^{14}\ M_\odot$
β	0.02
σ	$1400 \pm 200\ Km/s$
n	0.5

Note: d = cluster distance; L = cluster radius; $\theta_0 = L/d$; r_c = X-ray core radius; l_0 = central surface brightness; n_0 = central electron density; $x = h\nu/kT_r$; M_T = total cluster mass; $\beta = v_{clu}/c$; σ = line of sight velocity dispersion of the galaxies. n = power law index of the electron density profile.

Tab. 1b: Properties of the cluster Abell A576 (White & Silk 1980)

CLUSTER PROPERTIES	DATA
z	0.039
d	236 Mpc
L	$\sim$ 3 Mpc
θ_0	0.013
r_c	0.24 Mpc
l_0	3.8×10^{-5} erg cm^{-2} sec^{-1}
n_0	4.6×10^{-3} cm^{-3}
x	0.18 (ν = 11 GHz)
M_T	$(2.9 \pm 0.5) \times 10^{15}$ $M_\odot$
β	0.02
σ	1124 Km/s
n	0.64

Tab. 1c: Properties of the cluster Abell 2163 (Struble & Rood 1987; Arnaud et al. 1992; Elbaz et al. 1995)

CLUSTER PROPERTIES	DATA
z	0.201
d	1267 Mpc
L	$\sim$ 3 Mpc
θ_0	2.37×10^{-3}
r_c	0.305 $\pm$ 0.019 Mpc
l_0	2.16×10^{-3} erg cm^{-2} sec^{-1}
n_0	6.8×10^{-3} cm^{-3}
x	0.18 (ν = 11 GHz)
M_T	$(4.6 \pm 0.4) \times 10^{15}$ $M_\odot$
β	0.02
σ	$Unknown$
n	$0.62^{+0.05}_{-0.02}$

Tab. 2a: Different contributions to the CMB anisotropy and polarization in A2218.

F_1	F_2(**GB**)	F_2(**CO/EXP**)	F_3
1.63×10^{-6}	28.690	-2484.92 (**CO**)	
	0.116	-0.4720 (**SE**) -3.923×10^{-4}	
	0.116	0.142 (**EXP**)	

F_1: **Lensing contribution;**

F_2: **Gravitational bound contribution (GB); Gravitational collapse (CO) or expansion (Slight expansion (SE), Expansion (EXP)) contributions;**

F_3: **Sunyaev-Zel'dovich contribution (SZ); the Doppler shift contribution is negligible.**

The listed values correspond to the following choice of the cluster coordinates and properties: $r = 1063Mpc$, $\theta = 2.83 \times 10^{-3}\ rad$, $\phi = 0.785\ rad$, $\alpha = 1.570\ rad$.

Tab. 2b: Different contributions to the CMB anisotropy and polarization in A576.

F_1	F_2(**GB**)	F_2(**CO/EXP**)	F_3
5.89×10^{-6}	0.299 1.047×10^{-3} 1.047×10^{-3}	-125.62 (**CO**) -5.361×10^{-3} (**SE**) 1.342×10^{-3} (**EXP**)	-3.183×10^{-5}

F_1: **Lensing contribution;**

F_2: **Gravitational bound contribution (GB); Gravitational collapse (CO) or expansion (Slight expansion (SE), Expansion (EXP)) contributions;**

F_3: **Sunyaev-Zel'dovich contribution (SZ); the Doppler shift contribution is negligible.**

The listed values correspond to the following choice of the cluster coordinates and properties: $r = 239 Mpc$, $\theta = 0.013\ rad$, $\phi = 0.785\ rad$, $\alpha = 1.570\ rad$.

Tab. 2c: Different contributions to the CMB anisotropy and polarization in A2163.

F_1	$F_2(\mathbf{GB})$	$F_2(\mathbf{CO/EXP})$	F_3
9.56×10^{-6}	49.773	-3546.93 **(CO)**	
	0.206	-0.804 **(SE)** -5.411×10^{-5}	
	0.206	0.250 **(EXP)**	

F_1: **Lensing contribution;**

F_2: **Gravitational bound contribution (GB); Gravitational collapse (CO) or expansion (Slight expansion (SE), Expansion (EXP)) contributions;**

F_3: **Sunyaev-Zel'dovich contribution (SZ); the Doppler shift contribution is negligible.**

The listed values correspond to the following choice of the cluster coordinates and properties: $r = 1270 Mpc$, $\theta = 2.37 \times 10^{-3} \, rad$, $\phi = 0.785 \, rad$, $\alpha = 1.570 \, rad$.

Tab. 3: The coefficients of the expansion in Legendre Polynomial of the perturbed Stokes parameters: a_l refers to ΔI_l, b_l to ΔI_r while c_l, referring to ΔU, are zero for any l; the values have been calculated for a fixed r, respectively r=1063 Mpc for A2218, r=239 Mpc for A576 and r=1270 Mpc for A2163; I chose also $\theta = \theta_0$ and $\phi = \pi/4$.

CLUSTER	a_l	b_l
A2218	$a_0 = -1.206 \times 10^{+25}$ $a_1 = -7.638 \times 10^{+17}$ $a_2 \sim 0.0$	$b_0 = -7.654 \times 10^{+24}$ $b_1 = -7.636 \times 10^{+17}$ $b_2 \sim 0.0$
A576	$a_0 = -2.801 \times 10^{+24}$ $a_1 = -1.717 \times 10^{+17}$ $a_2 \sim 0.0$	$b_0 = -2.750 \times 10^{+24}$ $b_1 = -1.738 \times 10^{+17}$ $b_2 \sim 0.0$
A2163	$a_0 = -1.440 \times 10^{+25}$ $a_1 = 9.126 \times 10^{+17}$ $a_2 \sim 0.0$	$b_0 = -6.895 \times 10^{+24}$ $b_1 = 4.563 \times 10^{+17}$ $b_2 \sim 0.0$

FIGURE CAPTIONS

Fig. 1: The polarization ellipse (see also (Chandrasekhar 1960), p.26); the principal axes of the ellipse form the angles χ, $\chi + \pi/2$ with the direction $\vec{l}$; ψ is the angle formed by the generical vibration direction with $\vec{l}$. The Stokes parameters are defined as a function of I_l, I_r, χ and β (the tangent of β is the ratio of the axes of the ellipse traced by the end point of the electric vector) as in (Chandrasekhar 1960).

Fig. 2a: A moving lens produces a change in the brightness of an isotropic radiation field, proportional to the transverse velocity of the lens: the deflection causes a slight decrease in the photon energy if $\theta > 0$ and a slight increase if $\theta < 0$; in the Rayleigh-Jeans part of the CMB spectrum this effect appears as a brightness increase or decrease. The figure is taken from (Birkinshaw & Gull 1983).

Fig. 2b: The configuration of the problem in the frame of the observer; the lens has a velocity $\vec{v}$ in the (x, z) plane, while the angles (θ, ϕ) describe the direction of the deflected photon and respectively correspond to the angle between $\vec{k}$ and the z axis and to the angle between the projected $\vec{k}$ and the projected $\vec{v}$. The figure is taken from (Birkinshaw & Gull 1983).

Fig. 3 The average polarization degree as a function of the polar angle θ in the cluster **A2218**.

Fig. 4 The average polarization degree as a function of the polar angle θ in the cluster **A576**.

Fig. 5 The average polarization degree as a function of the polar angle θ in the cluster **A2163**.